

Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ be the points of the quadrilateral. We want to prove opposite midsegments are parallel.

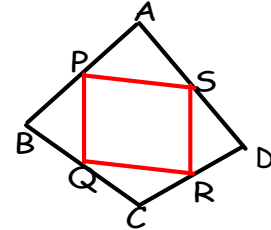
One side of our interior quadrilateral will connect the points P (M_{AB}) and S (M_{AD}), the other Q (M_{BC}) and R (M_{CD}). We need to prove that PS and QR have the same slope.

$$\begin{aligned} m_{PS} &= ((y_1+y_2)/2 - (y_1+y_4)/2) / ((x_1+x_2)/2 - (x_1+x_4)/2) \\ &= ((y_2-y_4)/2) / ((x_2-x_4)/2) \\ &= (y_2 - y_4) / (x_2 - x_4) \end{aligned}$$

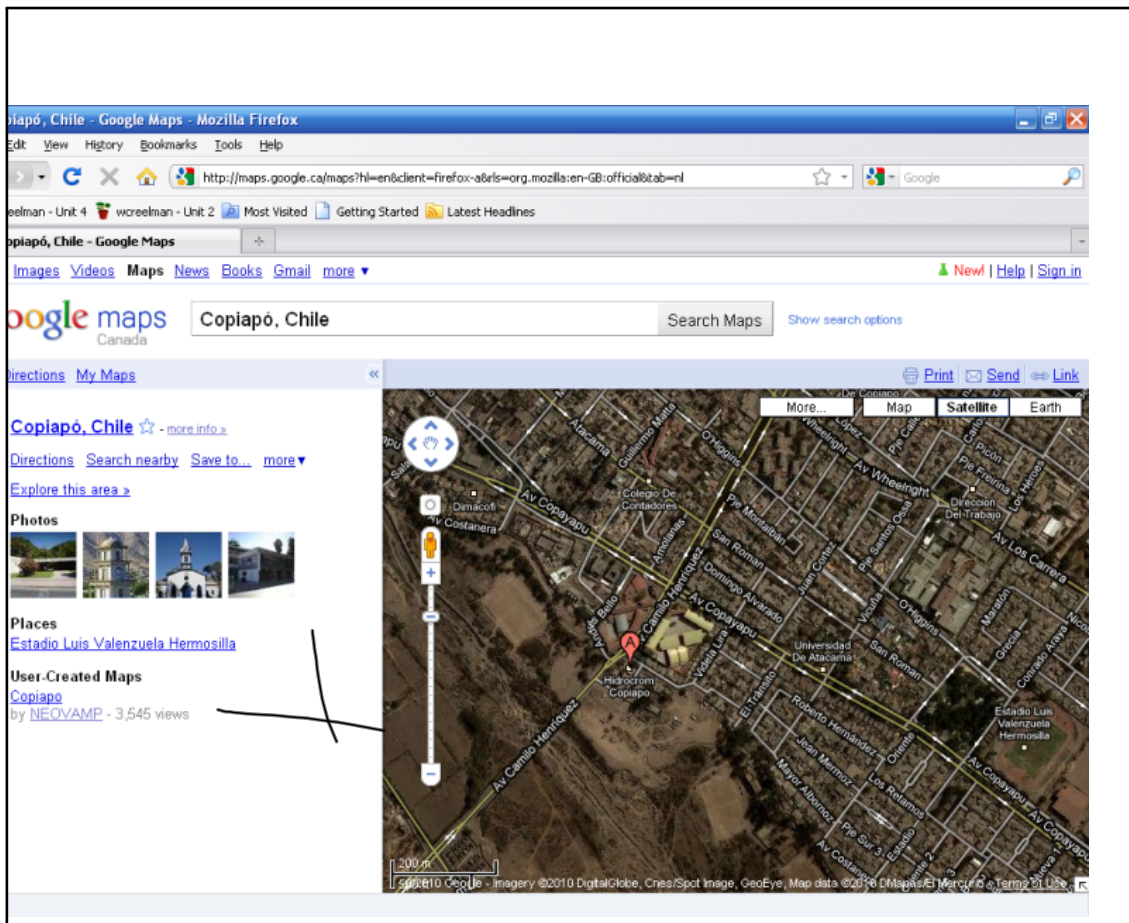
$$\begin{aligned} m_{QR} &= ((y_2+y_3)/2 - (y_3+y_4)/2) / ((x_2+x_3)/2 - (x_3+x_4)/2) \\ &= ((y_2-y_4)/2) / ((x_2-x_4)/2) \\ &= (y_2 - y_4) / (x_2 - x_4) \end{aligned}$$

As we can see the slopes are parallel.

Similarly PQ and SR can be shown to be parallel. Thus we have proved that the midpoints of the sides of any quadrilateral form a parallelogram



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